

28a) J (4P)

(since $\lambda^{(A)} = 3, \lambda^{(B)} = 5$)

b) D (4P)

(since $\lambda^{(A/B)} = 1 \pm 4i, y'_1 < 0$ if $y_1 = 0, y_2 > 0$)

c) F (4P)

(since $\lambda^{(A/B)} = 1 \pm 4i, y'_1 > 0$ if $y_1 = 0, y_2 > 0$)

d) C (4P)

(since $\lambda^{(A/B)} = -1 \pm 4i, y'_1 > 0$ if $y_1 = 0, y_2 > 0$)

e) E (4P)

(since $\lambda^{(A/B)} = -1 \pm 4i, y'_1 < 0$ if $y_1 = 0, y_2 > 0$)

f) I (4P)

(since $\lambda^{(A)} = -3, \lambda^{(B)} = -5$)

g) G (4P)

(since $\lambda^{(A/B)} = \pm 2, y'_1 > 0$ if $y_1 = 0, y_2 > 0$)

h) A (4P)

(since $\lambda^{(A/B)} = \pm 2i, y'_1 > 0$ if $y_1 = 0, y_2 > 0$)

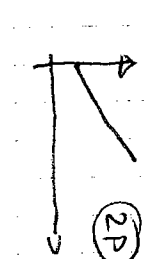
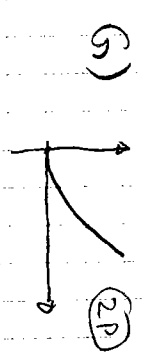
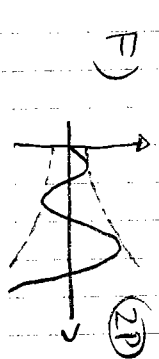
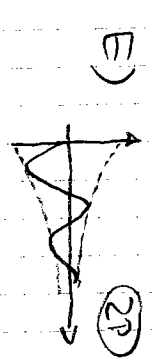
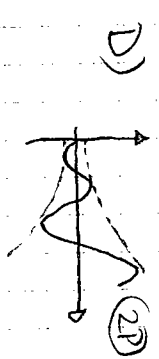
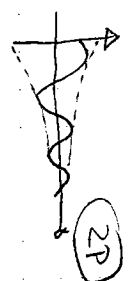
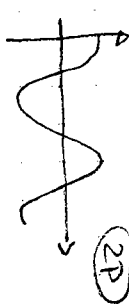
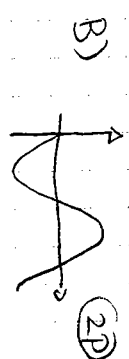
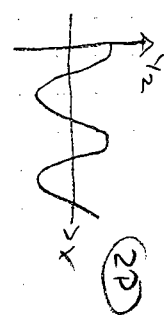
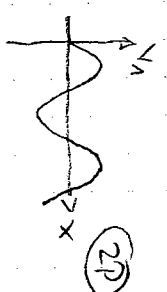
i) B (4P)

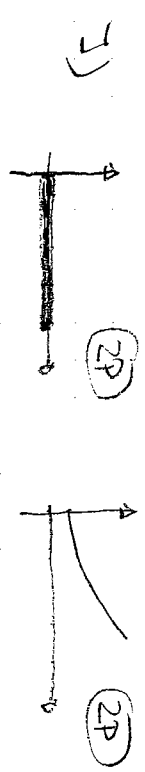
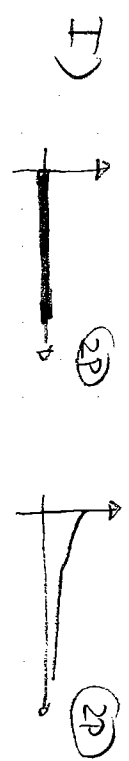
(since $\lambda^{(A/B)} = \pm 2i, y'_1 < 0$ if $y_1 = 0, y_2 > 0$)

j) H (4P)

(since $\lambda^{(A/B)} = \pm 2, y'_1 < 0$ if $y_1 = 0, y_2 > 0$)

29) A)





30a)

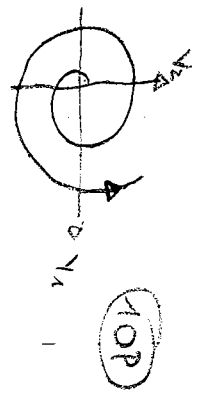
$$y_1' = e^x (\cos(x) - e^x \sin(x)) = y_1 - y_2$$

$$y_2' = e^x \sin(x) + e^x (\cos(x)) = y_1 + y_2$$

(40P)

b) $0 = (1 - \lambda)^2 + 1 = 0 \quad \lambda^{(A/B)} = 1 \pm i$

$x=0, y_1(0)=1$
 $y_2(0)=0$
 $y_2'(0)=1+0>0$

 (40P)

J) $y_2' = -y_2 = 0 \quad y_2 = C e^{-x}$ (4P) (lin. hom.)

$$y_1' = -y_1 + y_2 = -y_1 + C e^{-x} \quad (\text{lin. inhom.})$$

$$\Rightarrow y_1 = D e^{-x} + C x e^{-x}$$
 (4P)

$$y_2(0) = 1 = C \quad (6P) \quad y_1(0) = 1 = D \quad (6P)$$

Remark: $\begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$ has repeated eigenvalue $\lambda = -1$ and only one eigenvector $\vec{v} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (algebraic multiplicity, 2 $\neq$ geometric multiplicity, 1)

$\Rightarrow$ solution cannot be obtained by eigenvectors only (cf. textbook)